

The Rydberg Constant

The light you see when you plug in a hydrogen gas discharge tube is a shade of lavender, with some pinkish tint at a higher current. If you observe the light through a spectroscope, you can identify four distinct lines of color in the visible light range. The history of the study of these lines dates back to the late 19th century, where we meet a high school mathematics teacher from Basel, Switzerland, named Johann Balmer. Balmer created an equation describing the wavelengths of the visible hydrogen emission lines. However, he did not support his equation with a physical explanation. In a paper written in 1885, Balmer proposed that his equation could be used to predict the entire spectrum of hydrogen, including the ultra-violet and the infrared spectral lines. The Balmer equation is shown below.

$$\lambda = B \left(\frac{n^2}{n^2 - 4} \right)$$

where n is an integer greater than 2 (e.g., 3, 4, 5, or 6), and $B = 365.46$ nm. When one solves the equation, the calculated wavelengths are very close to the four emission lines in the visible light range for a hydrogen gas discharge tube. Balmer apparently derived his equation by trial and error. Sadly, he would not live to see Niels Bohr and Johannes Rydberg prove the validity of his equation.

Johannes Rydberg was a mathematics teacher like Balmer (he also taught a bit of physics). In 1890, Rydberg's research of spectroscopy (inspired, it is said, by the work of Dmitri Mendeleev) led to his discovery that Balmer's equation was a specific case of a more general principle. Rydberg substituted the wavenumber, $1/\text{wavelength}$, for wavelength and by applying appropriate constants he developed a variation of Balmer's equation. The Rydberg equation is shown below.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The energy levels are the integers in the equation, labeled n_i and n_f for initial and final levels, with R_H representing the Rydberg constant. Relating it to the Balmer equation, $R_H = 4/B$. For the hydrogen atom, n_f is 2, as shown in the first equation. The term $1/\lambda$ is the wavenumber, as expressed by Rydberg in his version of the Balmer equation. Niels Bohr used this equation to show that each line in the hydrogen spectrum corresponded to the release of energy by an electron as it passed from a higher to a lower energy level. In this experiment you will measure the emissions from a hydrogen gas discharge tube and analyze the emission data to calculate the Rydberg constant.

OBJECTIVES

In this experiment, you will

- Measure and analyze the emission spectrum of a hydrogen gas discharge tube.
- Use the data from the hydrogen emission spectrum to calculate the Rydberg constant.

MATERIALS

Ocean Optics Spectrometer
LabQuest or computer
LabQuest App or Logger *Pro*

Optical Fiber accessory
hydrogen gas discharge tube and power supply


Pre-Lab Exercise – Working with the Balmer Equation

Use the Balmer equation, as described in the introductory remarks, to calculate the four wavelengths in the visible light range for the hydrogen gas emission. Please record your information in the data table below. The calculated wavelengths should give you a strong clue as to the color of the four emission lines.

n_i	n_f	λ (nm)	color
	2		
	2		
	2		
	2		

PROCEDURE

1. Obtain and wear goggles.
2. Detach the cuvette holder/light source from the spectrometer. Connect the spectrometer to the USB port of LabQuest or a computer. Start the data-collection program, and then choose New from the File menu.
3. Connect a fiber optic cable to the threaded detector housing of the spectrometer.
4. To prepare the spectrometer for measuring light emissions: open the Experiment menu and select Change Units → Spectrometer: 1 → Intensity.
 - In LabQuest, from the Meter screen, tap Sensors and choose Change Units ► USB: Spectrometer ► Intensity.
 - In Logger *Pro*, open the Experiment menu and select Change Units ► Spectrometer: 1 ► Intensity.

5. To set an appropriate sampling time for collecting emission data:
 - a. In Logger *Pro*, open the Experiment menu and choose SetUp Sensors ► Spectrometer:1. In the small dialog that appears, change the Sample Time to 60 ms, change the Wavelength Smoothing to 0, and change the Samples to Average to 1.
 - b. In LabQuest, tap the Mode: Full Spectrum box on the Meter screen. On the Data Collection screen that will appear, change the Sample Time to 60 ms, change the Wavelength Smoothing to 0, and change the Samples to Average to 1.
6. Turn on the hydrogen gas discharge tube. Aim the tip of the fiber optic cable at the tube.
7. Start data collection. An emission spectrum will be graphed. Set the distance between the discharge tube and the tip of the fiber optic cable so that the peak intensity on the graph stays below 1.0. When you achieve a satisfactory graph, stop data collection.
8. To analyze your emission spectrum graph click the Examine icon, , on the toolbar in Logger *Pro*. In LabQuest, use your stylus or the left- and right-hand arrow keys to locate specific points on the graph. Identify as precisely as possible each of the four wavelengths of hydrogen's Balmer series. The third and fourth peaks are very small but they can be identified. Write down the four peaks of the graph in the data table below.
9. Store the run by tapping the File Cabinet icon in LabQuest, or by choosing Store Latest Run from the Experiment menu in Logger *Pro*.

NOTE: You will need to examine your graph of the hydrogen emissions to complete the Data Analysis section of the experiment.

DATA TABLE

Complete the table below. You will have recorded the wavelengths from examining the graph of the hydrogen discharge tube emissions.

Wavelength (nm)	Wavenumber (m^{-1})	Frequency (Hz)	Photon Energy (J)	n_i (Balmer Series)
				3
				4
				5
				6

Calculation Guide

- Wavelength: examine the graph and write down the peak in the specified regions.
- Wavenumber = $10^9 / (\text{wavelength in nm})$
- Frequency = $(3 \times 10^8 \text{ m/s}) / (\text{wavelength in m})$ **Note:** $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$
- Photon Energy = (frequency) $\times h$ **Note:** $h = 6.626 \times 10^{-34} \text{ m}^2 \cdot \text{kg} \cdot \text{s}^{-1}$

DATA ANALYSIS

1. Use the equation described in the introductory remarks to calculate the Rydberg constant for the four lines in Balmer Series that you identified in the table above. What is the average value for the Rydberg constant, based on your data?
2. A second method of determining the Rydberg constant is to analyze a graph of the values of n in the Balmer Series vs. the wavenumber. Prepare a plot of $1/n_i^2$ (X-values) vs. wavenumber (Y-values). Calculate the best-fit line (linear regression) equation for the plot; the slope of this line is equal to $-R_H$.
3. An accepted value of the Rydberg constant, R_H , is $1.097 \times 10^7 \text{ m}^{-1}$. Compare your value of R_m to the accepted value.
4. Use the R_H that you calculated in Question #3 to predict the wavelength of the fifth line in the Balmer Series ($n = 7$). Examine your graph of the hydrogen discharge tube emissions. Does the fifth Balmer line appear as a peak in your graph? Explain.